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Heavy Quark Effects in the Virtual Photon Structure Functions

Yoshio KITADONO[1], *) Ken SASAKI[2], **) Takahiro UEDA[3], ***)
and Tsuneo UEMATSU[4] †)

[1] *Dept. of Physics, Faculty of Science, Hiroshima University, Higashi Hiroshima 739-8526, Japan.*

[2] *Dept. of Physics, Faculty of Engineering, Yokohama National University, Yokohama 240-8501, Japan.*

[3] *High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan.*

[4] *Dept. of Physics, Graduate School of Science, Kyoto University, Yoshida, Kyoto 606-8501, Japan.*

Abstract

We investigate the heavy quark mass effects in the virtual photon structure functions $F_2^\gamma(x,Q^2,P^2)$ and $F_L^\gamma(x,Q^2,P^2)$ in the framework of the mass-independent renormalization group equation (RGE). We study a formalism in which the heavy quark mass effects are treated based on parton picture as well as on the operator product expansion (OPE), and perform the numerical evaluation of $F_{\rm eff}^\gamma(x,Q^2,P^2)$ to the next-leading order (NLO) in QCD.

*) e-mail: kitadono@scphys.kyoto-u.ac.jp
**) e-mail: sasaki@phys.ynu.ac.jp
***) e-mail: uedat@post.kek.jp
†) e-mail: uematsu@scphys.kyoto-u.ac.jp

§1. Introduction

The Large Hadron Collider (LHC)[1] has started its operation and it is anticipated that the signals for the new physics beyond the Standard Model (SM) will be discovered. Once these signals are observed, more precise measurements will need to be carried out at the future e^+e^- collider, so-called the International Linear Collider (ILC).[2] In such cases, it is still important for us to have detailed knowledge of the SM predictions at high energies based on QCD.

It is well known that, in high energy e^+e^- collision experiments, the cross section of the two-photon processes $e^+e^- \to e^+e^- +$ hadrons dominates over that of the one-photon annihilation processes $e^+e^- \to \gamma^* \to$ hadrons. The two-photon processes provide a good testing ground for studying the predictions of QCD at high energies. Here we consider the two-photon processes in the double-tag events where both of the outgoing e^+ and e^- are detected (see Fig. 1). In particular, we investigate the kinematical region in which one of the photons with momentum q is far off-shell (large $Q^2 \equiv -q^2$) while the other with momentum p is close to the mass-shell (small $P^2 = -p^2$), can be viewed as a deep-inelastic scattering where the target is a photon rather than a nucleon.[3] In this deep-inelastic scattering off photon targets, we can study the photon structure functions, which are the analogs of the nucleon structure functions.

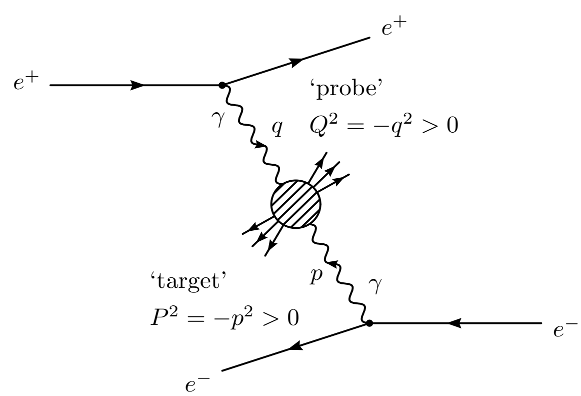

Fig. 1. Deep-inelastic scattering on a virtual photon in the e^+e^- collider experiments

The unpolarized (spin-averaged) photon structure functions $F_2^\gamma(x,Q^2)$ as well as $F_L^\gamma(x,Q^2)$ of the real photon ($P^2=0$) were first studied in the parton-model (PM)[4] and then investigated in perturbative QCD (pQCD). The leading order (LO) QCD contributions to $F_2^\gamma(x,Q^2)$ and $F_L^\gamma(x,Q^2)$ were obtained by Witten.[5] The next-to-leading order (NLO) corrections to $F_2^\gamma(x,Q^2)$ were calculated by Bardeen and Buras.[6] These results were obtained in the framework based on the operator product expansion (OPE)[7] and the renormalization group equation (RGE).[8] The same results were rederived by the QCD improved PM.[9]

The structure function $F_2^\gamma(x,Q^2,P^2)$ for the case of a virtual photon target ($P^2 \neq 0$) was investigated in the LO and in the NLO by pQCD.[10),11)] Also the virtual photon structure function $F_L^\gamma(x,Q^2,P^2)$ was studied in the LO.[11)] In fact, these structure functions were analyzed in the kinematical region, $\Lambda^2 \ll P^2 \ll Q^2$, where Λ is the QCD scale parameter. The advantage of studying a virtual photon target in this kinematical region is that we can calculate the whole structure function, its shape and magnitude, by the perturbative method. This is contrasted with the case of the real photon target where in the NLO there exist nonperturbative pieces. Recently the QCD analysis was made for $F_2^\gamma(x,Q^2,P^2)$ up to the next-to-next-to-leading order (NNLO) and for $F_L^\gamma(x,Q^2,P^2)$ up to the NLO.[12)] And more recently the target effects on these structure functions were studied[13)] and compared with the existing experimental data.[14),15)] In these calculations all the relevant quarks were assumed to be massless.

In this paper we examine the heavy quark mass effects on the photon structure functions $F_2^\gamma(x,Q^2,P^2)$ and $F_L^\gamma(x,Q^2,P^2)$. Indeed, the heavy quark mass effects for the two-photon processes, especially in the deep-inelastic kinematical region, have been studied by many authors.[16),17),18),19),20)] But they were not treated within the framework of the OPE and the RGE. Our analysis here is performed in the framework of the QCD improved PM powered by the parton evolution equations and based on the mass-independent renormalization group approach in which the RGE parameters, i.e., β and γ functions, are the same as those of the massless quark case. We consider the system which consists of $n_f - 1$ massless quarks and one heavy quark together with gluons and photons. Then, the heavy quark mass effects are included in the RGE inputs; the coefficient functions and the operator matrix elements. In the case of the nucleon target, the heavy quark mass effects were studied by a method based on the OPE in Ref. 21), where the heavy quark was treated such that it was radiatively generated and absent in the intrinsic quark components of the nucleon. This picture does not hold for the case of virtual photon target, since the heavy quark is also generated from the virtual photon target with light quarks at high energies. We should consider both the heavy and light quarks equally as the partonic components inside the virtual photon.

In the next section, we derive the evolution equations for the parton distribution functions in the case where $n_f - 1$ light quarks and one heavy quark are present. In section 3, we calculate the heavy quark effects in the virtual photon structure functions, $F_2^\gamma(x,Q^2,P^2)$, $F_L^\gamma(x,Q^2,P^2)$ and $F_{\rm eff}^\gamma(x,Q^2,P^2)$, and compare our theoretical predictions with the existing experimental data. The final section is devoted to the conclusions. We discuss the diagonalization of the anomalous dimension in Appendix A, and the parton-model derivation of the master formula in Appendix B.

§2. The evolution equations in the presence of the heavy quark effects

In this section we consider the evolution equations for the case in which $n_f - 1$ massless quarks and one heavy quark exist. The extension to the system with many heavy quarks is straightforward, since we can repeat this treatment recursively.

2.1. *Heavy quark effects and operator mixing*

We discuss the heavy quark effects in the RGE mixing and derive the master formula for the moments with quark mass effects in this subsection. Although the OPE is the useful formalism, we can get more physically intuitive picture in the PM approach, so we discuss the heavy quark effects in the evolution equations in the parton model. Now we consider the evolution equations for $n_f - 1$ massless quark parton distribution functions (PDFs) $q^i(x, Q^2, P^2)$ $(i = 1, \cdots, n_f - 1)$ and one heavy quark PDF $q^H(x, Q^2, P^2)$ together with the gluon PDF $G^\gamma(x, Q^2, P^2)$ and photon PDF $\Gamma^\gamma(x, Q^2, P^2)$, Experimentally, this situation corresponds to system of u,d,s (massless quarks) + c (heavy quark) for kinematical region of PLUTO data[14] and u,d,s,c (massless quarks) + b (heavy) for that of L3.[15] We write down the DGLAP equations for q^i, q^H, G^γ, Γ^γ;

$$\begin{aligned}
&\frac{dq^i(x, Q^2, P^2)}{d\ln Q^2} \\
&= \int_1^x \frac{dy}{y}\left[\sum_{j=1}^{n_f-1} \tilde{P}_{ij}\left(\frac{x}{y}, Q^2\right) q^j(y, Q^2, P^2) + \tilde{P}_{iH}\left(\frac{x}{y}, Q^2\right) q^H(y, Q^2, P^2)\right. \\
&\qquad\left. + \tilde{P}_{qG}\left(\frac{x}{y}, Q^2\right) G^\gamma(y, Q^2, P^2) + \tilde{P}_{i\gamma}\left(\frac{x}{y}, Q^2\right) \Gamma^\gamma(y, Q^2, P^2)\right], \qquad (2\cdot1)
\end{aligned}$$

$$\begin{aligned}
&\frac{dq^H(x, Q^2, P^2)}{d\ln Q^2} \\
&= \int_1^x \frac{dy}{y}\left[\sum_{j=1}^{n_f-1} \tilde{P}_{Hj}\left(\frac{x}{y}, Q^2\right) q^j(y, Q^2, P^2) + \tilde{P}_{HH}\left(\frac{x}{y}, Q^2\right) q^H(y, Q^2, P^2)\right. \\
&\qquad\left. + \tilde{P}_{HG}\left(\frac{x}{y}, Q^2\right) G^\gamma(y, Q^2, P^2) + \tilde{P}_{H\gamma}\left(\frac{x}{y}, Q^2\right) \Gamma^\gamma(y, Q^2, P^2)\right], \qquad (2\cdot2)
\end{aligned}$$

$$\begin{aligned}
&\frac{dG^\gamma(x, Q^2, P^2)}{d\ln Q^2} \\
&= \int_1^x \frac{dy}{y}\left[\sum_{j=1}^{n_f-1} \tilde{P}_{Gq}\left(\frac{x}{y}, Q^2\right) q^j(y, Q^2, P^2) + \tilde{P}_{GH}\left(\frac{x}{y}, Q^2\right) q^H(y, Q^2, P^2)\right. \\
&\qquad\left. + \tilde{P}_{GG}\left(\frac{x}{y}, Q^2\right) G^\gamma(y, Q^2, P^2) + \tilde{P}_{G\gamma}\left(\frac{x}{y}, Q^2\right) \Gamma^\gamma(y, Q^2, P^2)\right]. \qquad (2\cdot3)
\end{aligned}$$

where $\tilde{P}_{ij} = \delta_{ij}\tilde{P}_{qq} + \frac{1}{n_f}\tilde{P}^S_{qq}$ is the splitting functions of j-parton into i-parton, and the first term represents the process that j-quark splits into i-quark without through gluon, and the second term stands for the splitting through gluon and $\widetilde{P}_{qq}$ and $\widetilde{P}^S_{qq}$ are both independent of quark flavors, i and j. $\widetilde{P}^S_{qq}$ is relevant for the flavor-singlet part and starts in the order of α_s^2.

We now define the singlet combination, q_L^γ, and the non-singlet part of i-th quark q^i_{NS} as well as the non-singlet combination q^γ_{NS} for the light-flavors as,

$$q_L^\gamma \equiv \sum_{i=1}^{n_f-1} q^i, \quad q^i_{NS} \equiv q^i - \frac{1}{n_f-1} q_L^\gamma, \quad q^\gamma_{NS} \equiv \sum_{i=1}^{n_f-1} e_i^2 q^i_{NS}. \tag{2·4}$$

Note that $\sum_{i=1}^{n_f-1} q^i_{NS} = 0$. We should remember that the photon PDF in the virtual photon Γ^γ does not evolve within the order α_{QED}, so we set $\Gamma^\gamma(y, Q^2, P^2) = \delta(1-y)$. We can rewrite the Eqs.(2·1), (2·2) and (2·3) in terms of $q_L^\gamma, q^\gamma_{NS}$. This can be done by the following steps. At first, we sum up Eqs.(2·1),(2·2) and (2·3) from $i=1$ to $i=n_f-1$ and we get the equation for the parton distribution defined as a row vector $\boldsymbol{q}^\gamma(x, Q^2, P^2) = (q_L^\gamma, q^H, G^\gamma, q^\gamma_{NS})$,

$$\frac{d}{d\ln Q^2}\boldsymbol{q}^\gamma(x, Q^2, P^2) = \int_1^x \frac{dy}{y}\left[\boldsymbol{q}^\gamma(y, Q^2, P^2)\hat{P}\left(\frac{x}{y}, Q^2\right)\right] + \boldsymbol{k}(x, Q^2, P^2), \tag{2·5}$$

where the splitting functions are given by a matrix

$$\hat{P} \equiv \begin{pmatrix} P^S_{qq} & P_{LH} & P_{LG} & 0 \\ P_{HL} & P_{HH} & P_{HG} & 0 \\ P_{GL} & P_{GH} & P_{GG} & 0 \\ 0 & 0 & 0 & P^{NS}_{qq} \end{pmatrix} . \tag{2·6}$$

with

$$\begin{aligned} &P^S_{qq} = \tilde{P}_{qq} + \frac{n_f-1}{n_f}\tilde{P}^S_{qq}, \quad P_{LH} = \frac{n_f-1}{n_f}\tilde{P}^S_{qq}, \quad P_{LG} = (n_f-1)\tilde{P}_{qG}, \\ &P^S_{HL} = \frac{1}{n_f}\tilde{P}^S_{qq}, \quad P_{HH} = \tilde{P}_{qq} + \frac{1}{n_f}\tilde{P}^S_{qq}, \quad P_{HG} = \tilde{P}_{HG}, \\ &P^S_{GL} = \tilde{P}^S_{Gq}, \quad P_{GH} = \tilde{P}_{GH}, \quad P_{GG} = \tilde{P}_{GG}, \quad P^{NS}_{qq} = \tilde{P}_{qq} \ . \end{aligned} \tag{2·7}$$

and the inhomogeneous term, $\boldsymbol{k} \equiv (k_L, k_H, k_G, k_{NS})$ describing the parton-photon mixing is given by

$$k_L = \sum_{i=1}^{n_f-1}\tilde{P}_{i\gamma}, \quad k_H = \tilde{P}_{H\gamma}, \quad k_G = \tilde{P}_{G\gamma}, \quad k_{NS} = \sum_{i=1}^{n_f-1} e_i^2\left(\tilde{P}_{i\gamma} - \frac{1}{n_f-1}\sum_{j=1}^{n_f-1}\tilde{P}_{j\gamma}\right). \tag{2·8}$$

Note that the moments of the splitting functions $\tilde{P}_{ij}$ are related to the anomalous dimensions of operators $\gamma_n(g)$ and the coefficient function C^i_n satisfies the following mass-independent RGE:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma_m(g)m\frac{\partial}{\partial m}-\gamma_n(g,\alpha)\right]_{ij}C^j_n\left(\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2},\bar{g}(\mu^2),\alpha\right)=0\ , \qquad (2\cdot9)$$

where $\gamma_m(g)$ is the anomalous dimension for the mass operator. The solution to this equation is given by

$$C^i_n\left(\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2},\bar{g}(\mu^2),\alpha\right)=\left\{T\exp\left[\int^{\bar{g}(\mu^2)}_{\bar{g}(Q^2)}dg\frac{\gamma_n(g,\alpha)}{\beta(g)}\right]\right\}_{ij}C^j_n\left(1,\frac{\bar{m}^2}{Q^2},\bar{g}(Q^2),\alpha\right)\ , \qquad (2\cdot10)$$

where the anomalous dimension $\gamma_n(g,\alpha)$ is a 5×5 matrix and is given by

$$\gamma_n(g,\alpha)\equiv\begin{pmatrix}\hat{\gamma}_n & 0\\ \boldsymbol{K}_n & 0\end{pmatrix},\quad \hat{\gamma}_n\equiv\begin{pmatrix}\gamma^n_{LL} & \gamma^n_{HL} & \gamma^n_{GL} & 0\\ \gamma^n_{LH} & \gamma^n_{HH} & \gamma^n_{GH} & 0\\ \gamma^n_{LG} & \gamma^n_{HG} & \gamma^n_{GG} & 0\\ 0 & 0 & 0 & \gamma^n_{NS}\end{pmatrix}\ , \qquad (2\cdot11)$$

where

$$\boldsymbol{K}_n=(K^n_L,K^n_H,K^n_G,K^n_{NS})\ . \qquad (2\cdot12)$$

which describes the mixing between hadronic operators and the photon operator.

2.2. *Master formula for the moment*

We can summarize our master formula for the n-th moment of the virtual photon structure functions in the case where a heavy quark exists as

$$M^\gamma_2(n,Q^2,P^2)=\sum_{i,j=\psi,H,G,NS,\gamma}A^i_n\left(1,\frac{\bar{m}^2(P^2)}{P^2},\bar{g}(P^2)\right)\left\{T\exp\left[\int^{\bar{g}(P^2)}_{\bar{g}(Q^2)}dg\frac{\gamma_n(g,\alpha)}{\beta(g)}\right]\right\}_{ij}$$
$$\times C^j_{2,n}\left(1,\frac{\bar{m}^2(Q^2)}{Q^2},\bar{g}(Q^2)\right)\ , \qquad (2\cdot13)$$

where ψ (NS) is the flavor singlet (non-singlet) quark operators for the n_f-1 massless quarks. H stands for the heavy quark and $\bar{m}(Q^2)$ is the running mass evaluated at Q^2. A^i_n is the operator matrix element renormalized at $\mu^2=P^2$, while $C^i_{2,n}$ is the coefficient function renormalized at $\mu^2=Q^2$. Since $A^i_n,C^i_{2,n}$ are the solutions of the renormalization group equation, they depend on the running masses $\bar{m}(Q^2)$ and $\bar{m}(P^2)$. We can decompose the

moments Eq. (2·13) into that for the massless case and the additional term $\mathit{\Delta} M_2^\gamma(n,Q^2,P^2)$ due to the mass effects:

$$M_2^\gamma(n,Q^2,P^2)=M_2^\gamma(n,Q^2,P^2)\Big|_{\rm massless}+\mathit{\Delta} M_2^\gamma(n,Q^2,P^2)\ . \qquad (2\cdot 14)$$

They are obtained up to NLO by the diagonalization of the anomalous dimension matrix, which will be discussed in Appendix A.

The master formula for the n-th moment to NLO is given by

$$\begin{aligned}
&M_2^\gamma(n,Q^2,P^2)=\int_0^1 dx x^{n-2}F_2^\gamma(x,Q^2,P^2)\\
&=\frac{\alpha}{4\pi}\frac{1}{2\beta_0}\left[\frac{4\pi}{\alpha_s(Q^2)}\sum_i\mathcal{L}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right]+\sum_i\mathcal{A}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n}\right]\right.\\
&\qquad\left.+\sum_i\mathcal{B}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right]+\mathcal{C}^n\right]+\mathcal{O}(\alpha_s)\ ,
\end{aligned}\qquad (2\cdot 15)$$

where α (α_s) is the QED (QCD) coupling constant. The summation index i runs over $\pm, NS$ corresponding to the three eigenvalues $\lambda_\pm^n$ and λ_{NS}^n of the one-loop anomalous dimension matrix in the massless case. While for the present case with a heavy flavor, i runs over $\psi,\pm,NS$ for the four eigenvalues of the one-loop anomalous dimension matrix $\hat{\gamma}_n^{(0)}$ which turns out to be λ_ψ^n, $\lambda_\pm^n$, λ_{NS}^n, where we have $\lambda_\psi^n=\lambda_{NS}^n$ (See Appendix A). For the notation of the renormalization group parameters we refer to Ref. 12). The LO coefficients $\mathcal{L}_i^n$, the NLO coefficients $\mathcal{A}_i^n,\mathcal{B}_i^n,\mathcal{C}^n$ are,

$$\begin{aligned}
\mathcal{L}_i^n&=\boldsymbol{K}_n^{(0)}P_i^n\boldsymbol{C}_{2,n}^{(0)}\frac{1}{1+d_i^n},\\
\mathcal{A}_i^n&=-\boldsymbol{K}_n^{(0)}\sum_j\frac{P_j^n\hat{\gamma}_n^{(1)}P_i^n}{\lambda_j^n-\lambda_i^n+2\beta_0}\boldsymbol{C}_{2,n}^{(0)}\frac{1}{d_i^n}-\boldsymbol{K}_n^{(0)}P_i^n\boldsymbol{C}_{2,n}^{(0)}\frac{\beta_1}{\beta_0}\frac{1-d_i^n}{d_i^n}\\
&\quad+\boldsymbol{K}_n^{(1)}P_i^n\boldsymbol{C}_{2,n}^{(0)}\frac{1}{d_i^n}-2\beta_0\tilde{\boldsymbol{A}}_n^{(1)}P_i^n\boldsymbol{C}_{2,n}^{(0)},\\
\mathcal{B}_i^n&=\boldsymbol{K}_n^{(0)}\sum_j\frac{P_i^n\hat{\gamma}_n^{(1)}P_j^n}{\lambda_i^n-\lambda_j^n+2\beta_0}\boldsymbol{C}_{2,n}^{(0)}\frac{1}{1+d_i^n}+\boldsymbol{K}_n^{(0)}P_i^n\boldsymbol{C}_{2,n}^{(1)}\frac{1}{1+d_i^n}\\
&\quad-\boldsymbol{K}_n^{(0)}P_i^n\boldsymbol{C}_{2,n}^{(0)}\frac{\beta_1}{\beta_0}\frac{d_i^n}{1+d_i^n},\\
\mathcal{C}^n&=2\beta_0\left(C_{2,n}^{\gamma(1)}+\tilde{\boldsymbol{A}}_n^{(1)}\cdot\boldsymbol{C}_{2,n}^{(0)}\right).
\end{aligned}\qquad (2\cdot 16)$$

where $\boldsymbol{K}_n^{(0)}$ ($\boldsymbol{K}_n^{(1)}$) is the 1-loop (2-loop) photon-parton mixing anomalous dimension, P_i^n's are projection operators, $\boldsymbol{C}_{2,n}^{(0)}$ ($\boldsymbol{C}_{2,n}^{(1)}$) is the tree-level (1-loop) coefficient function and β_0

(β_1) is the 1-loop (2-loop) beta function. $d_i^n = \lambda_i^n/2\beta_0$. $\boldsymbol{A}_n^{(1)} = (\alpha/4\pi)\tilde{\boldsymbol{A}}_n^{(1)}$ is the 1-loop operator matrix element and $C_{2,n}^{\gamma(1)}$ is the coefficient function of the photon operator. (See the Ref. 12) for details). We can reorganize the summation over $i = \psi, \pm, NS$ for the case with a heavy flavor, into that for $i = \pm, NS$ since the eigenvalue λ_ψ^n gives rise to the same exponents in Eq. (2·15), as $\lambda_\psi^n = \lambda_{NS}^n$. The additional terms arising from the variation of OME and the coefficient functions due to the heavy quark effects are,

$$
\begin{aligned}
\Delta M_2^\gamma(n, Q^2, P^2, m^2) &= \int_0^1 dx x^{n-2} \Delta F_2^\gamma(x, Q^2, P^2, m^2) \\
= \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\Bigg[&\sum_{i=\pm,NS} \Delta\mathcal{A}_i^n \left[1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n}\right] \\
&+ \sum_{i=\pm,NS} \Delta\mathcal{B}_i^n \left[1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right] + \Delta\mathcal{C}^n \Bigg] + \mathcal{O}(\alpha_s) ,
\end{aligned} \tag{2·17}
$$

where $\Delta\mathcal{A}_i^n, \Delta\mathcal{B}_i^n, \Delta\mathcal{C}^n$ are the deviations from the massless case due to the heavy quark effects. Note that the heavy quark effects do not change the LO coefficients $\mathcal{L}_i^n$ (See Appendix A). This can be derived by an alternative method (See Appendix B). The explicit expressions of $\Delta\mathcal{A}_i^n, \Delta\mathcal{B}_i^n, \Delta\mathcal{C}^n$ are

$$
\begin{aligned}
\Delta\mathcal{A}_{NS}^n &= \frac{1}{n_f} e_H^2 (-12\beta_0) \Delta\tilde{A}_{nG}^\psi (e_H^2 - \langle e^2\rangle_{n_f}), \\
\Delta\mathcal{A}_\pm^n &= \frac{1}{n_f} e_H^2 (-12\beta_0) \langle e^2\rangle_{n_f} \Delta\tilde{A}_{nG}^\psi \frac{\gamma_{\psi\psi}^{0,n} - \lambda_\mp^n}{\lambda_\pm^n - \lambda_\mp^n}, \\
\Delta\mathcal{B}_{NS}^n &= 24\frac{n^2+n+2}{n(n+1)(n+2)} e_H^2 (e_H^2 - \langle e^2\rangle_{n_f}) \frac{1}{1+d_{NS}^n} \Delta B_\psi^n, \\
\Delta\mathcal{B}_\pm^n &= 24\frac{n^2+n+2}{n(n+1)(n+2)} e_H^2 \langle e^2\rangle_{n_f} \frac{1}{1+d_\pm^n} \\
&\quad \times \left[\frac{\gamma_{\psi\psi}^{0,n} - \lambda_\mp^n}{\lambda_\pm^n - \lambda_\mp^n}\Delta B_\psi^n + \frac{\gamma_{G\psi}^{0,n}}{\lambda_\pm^n - \lambda_\mp^n}\Delta B_G^n\right], \\
\Delta\mathcal{C}^n &= \frac{1}{n_f} 12\beta_0 e_H^2 \left(\Delta B_G^n + \Delta\tilde{A}_{nG}^\psi\right).
\end{aligned} \tag{2·18}
$$

where e_H is the heavy quark charge,

$$
e_H = \begin{cases} +\frac{2}{3}, & \text{for } SU(2)_L \text{ up-type quark}, \\ -\frac{1}{3}, & \text{for } SU(2)_L \text{ down-type quark}. \end{cases} \tag{2·19}
$$

Here we have adopted the notation of Ref. 6) for the anomalous dimensions and coefficient functions in the $\overline{\text{MS}}$ scheme,[22] and $\Delta\tilde{\boldsymbol{A}}_n^{(1)} = 6(\langle e^2\rangle, 0, \langle e^4\rangle - \langle e^2\rangle^2)\Delta\tilde{A}_{nG}^\psi$.

Thus there is no change for the moment at LO level. This is explained by the following discussion. In terms of our approach, the heavy quark effect is included in the RGE inputs; OME and the coefficient functions. The evolution factor is the same as the massless case and there is no physical difference which distinguishes the quarks except for the electric charge in the RGE inputs at this order (LO). This results is also justified by the explicit calculation for the LO moment. So, the heavy quark effect is occurred at NLO level. Therefore the higher order(more than NLO) calculation is essential in the case of including the heavy quark effects with massless calculations for the moments. The variation of coefficients $\Delta\mathcal{A}_i^n, \Delta\mathcal{B}_i^n, \Delta\mathcal{C}^n$ can be obtained by considering the possible variation of the coefficient functions and the operator matrix element at NLO.

Here we confine ourselves to the case of the limit: $\Lambda^2_{\rm QCD} \ll P^2 \ll m^2 \ll Q^2$. In this limit we have

$$\Delta\tilde{A}^{\psi}_{nG}\frac{1}{n_f} = 2\left[-\frac{n^2+n+2}{n(n+1)(n+2)}\ln\frac{m^2}{P^2}+\frac{1}{n}-\frac{1}{n^2}\right.$$
$$\left.+\frac{4}{(n+1)^2}-\frac{4}{(n+2)^2}-\frac{n^2+n+2}{n(n+1)(n+2)}\sum_{j=1}^{n}\frac{1}{j}\right] , \qquad (2\cdot20)$$

$$\Delta B^n_{\psi}=0, \quad \Delta B^n_G = 0, \quad \Delta B^n_{\gamma} = 2\Delta B^n_G/n_f = 0. \qquad (2\cdot21)$$

The quark (gluon) coefficient functions B^n_{ψ} (B^n_G) are obtained by taking the difference between photon-parton amplitude and the operator matrix elements.[22),23)] In the large mass limit $P^2 \ll m^2$, the deviations from the massless case are the same for the photon-parton amplitudes and the operator matrix elements. Hence we have $\Delta B^n_{\psi} = \Delta B^n_G = 0$. We will discuss the details elsewhere.[24)] We also note here for the longitudinal structure function $F^{\gamma}_L(x,Q^2,P^2)$ we do not have heavy quark mass effects to the LO ($\mathcal{O}(\alpha)$) and is given by the same formula as the massless case.

§3. Numerical analysis of $F^{\gamma}_{\rm eff}(x,Q^2,P^2)$

The virtual photon structure functions are recovered from the moments by the inverse Mellin transformation. In this section we examine the heavy quark mass effects on the effective photon structure function[25)] $F^{\gamma}_{\rm eff}(x,Q^2,P^2)$ defined as

$$F^{\gamma}_{\rm eff}(x,Q^2,P^2) = F^{\gamma}_2(x,Q^2,P^2)+\frac{3}{2}F^{\gamma}_L(x,Q^2,P^2) \ . \qquad (3\cdot1)$$

We evaluate $F^{\gamma}_{\rm eff}$ up to the NLO and compare our theoretical predictions with the existing experimental data from PLUTO Collaboration[14)] and L3 Collaboration.[15)] For the PLUTO (L3) data, we have $Q^2 = 5$ (120) GeV^2 and $P^2 = 0.35$ (3.7) GeV^2. Therefore, we assume that

the active flavors are u, d, s (massless) plus c (heavy) for the case of PLUTO and u, d, s, c (massless) plus b (heavy) for L3.

We plot the experimental data from PLUTO group in Fig. 2 and those from L3 group in Fig 3, together with our theoretical predictions. We also show the curves of the NLO predictions when active quarks are all massless. We use the QCD running quark mass $\overline{m}(P^2)$ which is valid up to the NLO[26)] and we adopt the following values of the quark masses as inputs,[27)]

$$m_c = 1.3\text{GeV} \quad \text{(for PLUTO)}, \tag{3·2}$$

$$m_b = 4.2\text{GeV} \quad \text{(for L3)}. \tag{3·3}$$

In general the heavy quark mass has an effect of reducing the photon structure functions in magnitude. This feature is explained by the suppression of the heavy quark production rate due to the existence of their masses. Heavy quark mass effects appear at larger x region. Due to the kinematical constraint for the heavy quark production $(p+q)^2 \geq 4m^2$, the contribution of heavy quark to the structure functions exists below $x_{\max} = \frac{1}{1+\frac{4m^2}{Q^2}}$ and, therefore, the difference between the massless and the massive cases emerges above $x_{\max}$. This kinematical "threshold" effect is not clearly seen in our analysis since we adopted the framework based on the OPE and took into account only the leading twist-2 operators. But still we see that the difference between the massless and the massive cases becomes bigger at large x (see Fig.2 and Fig.4 below). It is also noted that the heavy quark mass effects are sensitive to the electric charge of the relevant quark. Since the photon structure functions depend on the quark-charge factors $\langle e^2\rangle$ and $\langle e^4\rangle$, the up-type heavy quark gives larger contribution to the photon structure functions than the down-type quark.

In the case of PLUTO data, there is no justification of our approximation for the limit $Q^2 \gg m^2$. But we find in Fig. 2 that the predicted curve with mass effects shows the trend of reducing the "over-estimated" massless QCD calculation, especially at larger x region, and appears to be closer to the experimental data. For L3 data, the hierarchical condition $P^2 \ll m^2 \ll Q^2$ is satisfied. Although the experimental error bars are rather large, we find in Fig. 3 that theoretical curves, both massive and massless cases, are roughly consistent with the data, except for the larger x region. For the L3 region, the heavy quark mass effects are almost negligible since the b has a charge $-1/3$.

Finally we present, in Fig. 4, our prediction for the case $Q^2 = 30\text{GeV}^2$ with $P^2 = 0.35\text{GeV}^2$, as an illustration when massive charm quark is relevant. The condition $P^2 \ll m^2 \ll Q^2$ is satisfied. Although there is no experimental data corresponding to this case, we find that the heavy quark mass effects are sizable as can be seen in the figure.

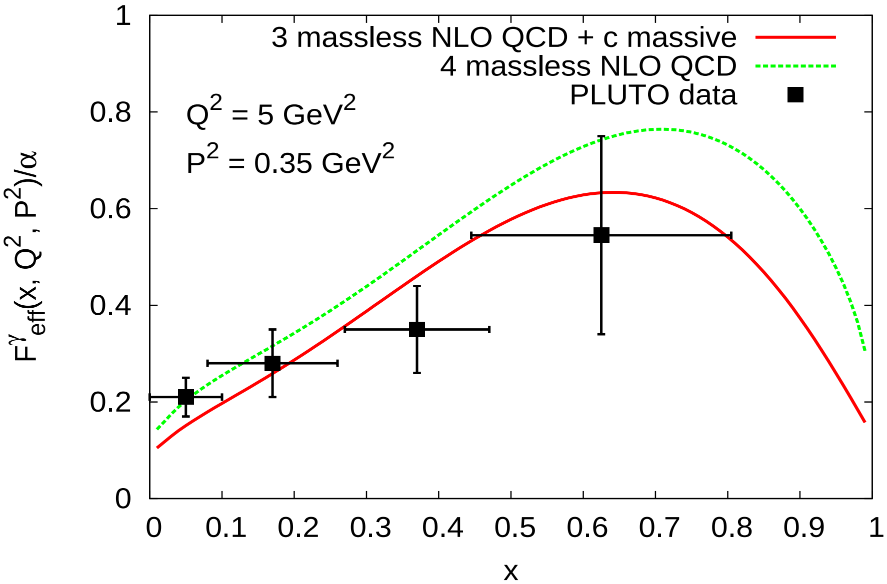

Fig. 2. $F^{\gamma}_{\rm eff}$ to NLO in QCD and PLUTO data. $n_f = 4$, $Q^2 = 5\mathrm{GeV}^2$, $P^2 = 0.35\mathrm{GeV}^2$, $x_{\max} = 0.43$.

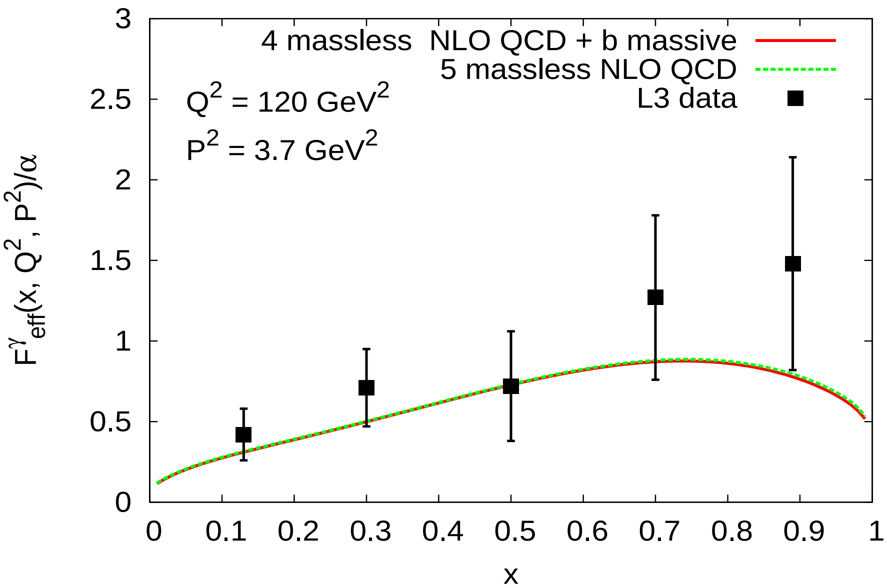

Fig. 3. $F^{\gamma}_{\rm eff}$ to NLO in QCD and L3 data. $n_f = 5$, $Q^2 = 120\mathrm{GeV}^2$, $P^2 = 3.7\mathrm{GeV}^2$,$x_{\max} = 0.63$.

§4. Conclusions

We have investigated the heavy quark mass effects in the virtual photon structure function based on the parton picture as well as on the operator product expansion. We have derived the master formula for the additional contributions at NLO to the moments due to mass effects. The heavy quark mass effect does not change the LO moments of the photon structure

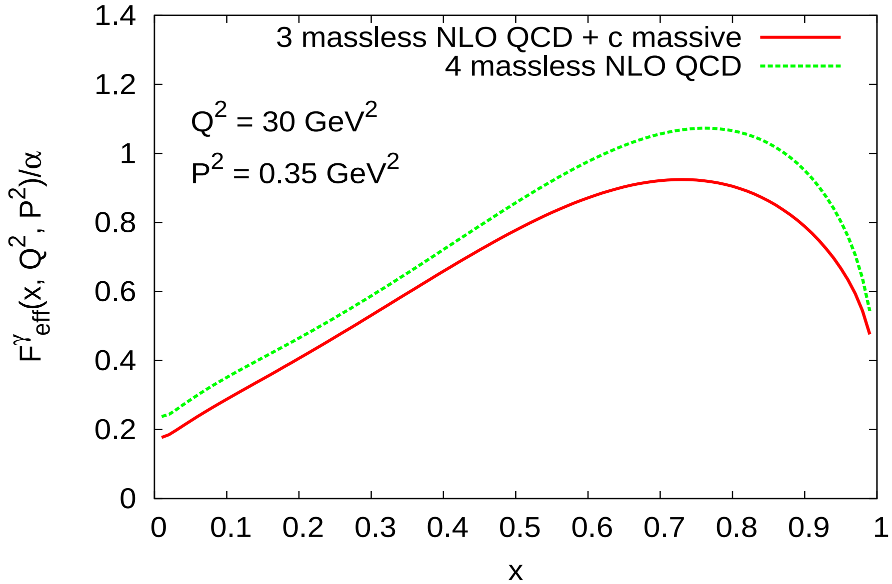

Fig. 4. $F_{\rm eff}^{\gamma}$ to NLO in QCD for $n_f = 4$ $Q^2 = 30{\rm GeV}^2$ with $P^2 = 0.35{\rm GeV}^2$, $x_{\rm max} = 0.82$.

functions but it changes NLO moments. We applied this formalism to the phenomenological analysis of $F_{\rm eff}^{\gamma}(x, Q^2, P^2)$. We confronted the theoretical QCD prediction to NLO including the heavy quark mass effects with the existing experimental data.

For the kinematical region of the PLUTO data we assumed the 3 quarks (u,d,s) are massless, while the charm quark (c) is the heavy quark. In the region for the L3 data we took the 4 quarks (u,d,s,c) to be massless, and the bottom (b) quark to be the heavy quark. For the PLUTO region, there exists a sizable heavy quark effect at larger x regime. Although our approximation assuming $m^2 \ll Q^2$ is not immediately applicable for the PLUTO region, the theoretical prediction shows a right trend of describing the experimental data. For the L3 region, the heavy quark mass effects are almost negligible since the b has a charge $-1/3$. It is somewhat remarkable that the theoretical predictions for the both cases are consitent with experimental data as for the total normalization. We only have one adjustable parameter, $\Lambda_{\rm QCD}$, which we took 0.2 GeV.

It would be interesting to investigate the charm and bottom quark mass effects at the future SUPER-B experiments.[29] If the center of mass energy of the future linear collider (ILC) is enough to produce the top quarks, its mass effects would be important for the measurements of the virtual photon structure functions in view of the charge-factor enhancement.

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Appendix A

—— *Diagonalization of the anomalous dimension matrix* ——

We briefly describe the diagonalization of the anomalous dimension matrix (2·11). Expanding the anomalous dimension in a power series of the coupling constant g:

$$\hat{\gamma}(g) = \frac{g^2}{16\pi^2}\hat{\gamma}_n^{(0)} + \frac{g^4}{(16\pi^2)^2}\hat{\gamma}_n^{(1)} + \cdots \ , \tag{A·1}$$

and decomposing the one-loop anomalous dimension matrix $\hat{\gamma}_n^{(0)}$ as a block matrix

$$\hat{\gamma}_n^{(0)} = \left(\begin{array}{c|c} \widetilde{\gamma}_n^{(0)} & 0 \\ \hline 0 & \gamma_{NS}^{0,n} \end{array}\right) , \tag{A·2}$$

with the $\widetilde{\gamma}_n^{(0)}$ expressed in terms of one-loop anomalous dimensions[22)]

$$\widetilde{\gamma}_n^{(0)} = \begin{pmatrix} \gamma_{\psi\psi}^{0,n} & 0 & \gamma_{G\psi}^{0,n} \\ 0 & \gamma_{\psi\psi}^{0,n} & \gamma_{G\psi}^{0,n} \\ \frac{n_f-1}{n_f}\gamma_{\psi G}^{0,n} & \frac{1}{n_f}\gamma_{\psi G}^{0,n} & \gamma_{GG}^{0,n} \end{pmatrix} , \tag{A·3}$$

we get the eigenvalues of the above 3×3 matrix, $\lambda^n = \lambda_\psi^n, \lambda_+^n, \lambda_-^n$ given by

$$\lambda_\psi^n = \gamma_{\psi\psi}^{0,n}, \ \lambda_\pm^n = \frac{1}{2}\left\{\gamma_{\psi\psi}^{0,n} + \gamma_{GG}^{0,n} \pm \sqrt{(\gamma_{\psi\psi}^{0,n} - \gamma_{GG}^{0,n})^2 + 4\gamma_{\psi G}^{0,n}\gamma_{G\psi}^{0,n}}\right\} . \tag{A·4}$$

Introducing the projection operators we can write down $\widetilde{\gamma}_n^{(0)}$ as

$$\widetilde{\gamma}_n^{(0)} = \sum_{i=\psi,\pm} \lambda_i^n P_i^n \ , \tag{A·5}$$

where the projection operators are given as

$$P_\psi^n = \begin{pmatrix} \frac{1}{n_f} & -\frac{1}{n_f} & 0 \\ -\frac{n_f-1}{n_f} & \frac{n_f-1}{n_f} & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{A·6}$$

$$P_\pm^n = \frac{1}{\lambda_\pm^n - \lambda_\mp^n}\begin{pmatrix} \frac{n_f-1}{n_f}(\gamma_{\psi\psi}^{0,n} - \lambda_\mp^n) & \frac{1}{n_f}(\gamma_{\psi\psi}^{0,n} - \lambda_\mp^n) & \gamma_{G\psi}^{0,n} \\ \frac{n_f-1}{n_f}(\gamma_{\psi\psi}^{0,n} - \lambda_\mp^n) & \frac{1}{n_f}(\gamma_{\psi\psi}^{0,n} - \lambda_\mp^n) & \gamma_{G\psi}^{0,n} \\ \frac{n_f-1}{n_f}\gamma_{\psi G}^{0,n} & \frac{1}{n_f}\gamma_{\psi G}^{0,n} & \gamma_{GG}^{0,n} - \lambda_\mp^n \end{pmatrix} . \tag{A·7}$$

Including flavor non-singlet anomalous dimension γ^n_{NS} into the 4×4 matrix (A·2), the projection operators are extended to

$$\hat{\gamma}^{(0)}_n = \sum_{i=\psi,+,-,NS} \lambda^n_i P^n_i \ , \tag{A·8}$$

where

$$P^n_\psi = \left(\begin{array}{ccc|c} \frac{1}{n_f} & -\frac{1}{n_f} & 0 & 0 \\ -\frac{n_f-1}{n_f} & \frac{n_f-1}{n_f} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right) , \tag{A·9}$$

$$P^n_\pm = \frac{1}{\lambda^n_\pm - \lambda^n_\mp} \left(\begin{array}{ccc|c} \frac{n_f-1}{n_f}(\gamma^{0,n}_{\psi\psi} - \lambda^n_\mp) & \frac{1}{n_f}(\gamma^{0,n}_{\psi\psi} - \lambda^n_\mp) & \gamma^{0,n}_{G\psi} & 0 \\ \frac{n_f-1}{n_f}(\gamma^{0,n}_{\psi\psi} - \lambda^n_\mp) & \frac{1}{n_f}(\gamma^{0,n}_{\psi\psi} - \lambda^n_\mp) & \gamma^{0,n}_{G\psi} & 0 \\ \frac{n_f-1}{n_f}\gamma^{0,n}_{\psi G} & \frac{1}{n_f}\gamma^{0,n}_{\psi G} & \gamma^{0,n}_{GG} - \lambda^n_\mp & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right) , \tag{A·10}$$

$$P^n_{NS} = \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) . \tag{A·11}$$

Now let us see that the leading order coefficients $\mathcal{L}_i$ are not changed in the presence of the heavy quark mass effects. With the four eigenvalues, the leading-order coefficients $\widehat{\mathcal{L}}_i$ turn out to be

$$\widehat{\mathcal{L}}^n_\psi = 24 \frac{n^2+n+2}{n(n+1)(n+2)} \frac{n_f-1}{n_f} (\langle e^2\rangle_{n_f-1} - e^2_H)^2 \frac{1}{1+d^n_\psi} \ , \tag{A·12}$$

$$\begin{aligned} \widehat{\mathcal{L}}^n_\pm = 24 &\frac{n^2+n+2}{n(n+1)(n+2)} \frac{1}{\lambda^n_\pm - \lambda^n_\mp} \frac{1}{n_f} \left\{(n_f-1)\langle e^2\rangle_{n_f-1} + e^2_H\right\}^2 \\ &\times(\gamma^{(0,n)}_{\psi\psi} - \lambda^n_\mp) \frac{1}{1+d^n_\pm} \ , \end{aligned}$$

$$\widehat{\mathcal{L}}^n_{NS} = 24 \frac{n^2+n+2}{n(n+1)(n+2)} (n_f-1)(\langle e^2\rangle_{n_f-1} - e^2_H)^2 \frac{1}{1+d^n_{NS}} \ , \tag{A·13}$$

where we have denoted the LO coefficients with a hat for the massive case in order to distinguish them from those for the massless case. Hence the leading-order QCD result for the moment of F^γ_2 is given by

$$\int_0^1 dx\, x^{n-2} F^\gamma_2(x, Q^2, P^2)$$

$$= \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\sum_{i=\psi,\pm,NS}\widehat{\mathcal{L}}_i^n\frac{4\pi}{\alpha_s(Q^2)}\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right] \ . \tag{A·14}$$

Here we note that

$$\lambda_\psi = \lambda_{NS} = \gamma_{\psi\psi}^{0,n}, \quad d_\psi^n = d_{NS}^n = \gamma_{\psi\psi}^{0,n}/2\beta_0 \ . \tag{A·15}$$

So we find the sum of $\widehat{\mathcal{L}}_\psi^n$ and $\widehat{\mathcal{L}}_{NS}^n$:

$$\begin{aligned}\widehat{\mathcal{L}}_\psi^n + \widehat{\mathcal{L}}_{NS}^n &= 24\frac{n^2+n+2}{n(n+1)(n+2)}\frac{1}{1+d_\psi^n} \\ &\times\left\{\frac{n_f-1}{n_f}(\langle e^2\rangle_{n_f-1}-e_H^2)^2 \ , +(n_f-1)\langle e^4\rangle_{n_f-1}-(n_f-1)\langle e^2\rangle_{n_f-1}^2\right\} \ .\end{aligned} \tag{A·16}$$

Now let us remind the following relations:

$$\langle e^2\rangle_{n_f-1} \equiv \frac{1}{n_f-1}\sum_{i=1}^{n_f-1} e_i^2, \quad \langle e^4\rangle_{n_f-1} \equiv \frac{1}{n_f-1}\sum_{i=1}^{n_f-1} e_i^4, \tag{A·17}$$

then we have

$$\begin{aligned}\widehat{\mathcal{L}}_\psi^n + \widehat{\mathcal{L}}_{NS}^n &= 24\frac{n^2+n+2}{n(n+1)(n+2)}\frac{1}{1+d_\psi^n} \\ &\times\left\{\sum_{i=1}^{n_f}e_i^4-\frac{1}{n_f}\left(\sum_{i=1}^{n_f}e_i^2-e_H^2\right)^2-\frac{2}{n_f}e_H^2\left(\sum_{i=1}^{n_f}e_i^2-e_H^2\right)-\frac{1}{n_f}e_H^4\right\} \\ &= 24\frac{n^2+n+2}{n(n+1)(n+2)}\frac{1}{1+d_\psi^n}\left\{\sum_{i=1}^{n_f}e_i^4-\frac{1}{n_f}\left(\sum_{i=1}^{n_f}e_i^2\right)^2\right\} = \mathcal{L}_{NS} \ .\end{aligned} \tag{A·18}$$

We also find $\widehat{\mathcal{L}}_\pm = \mathcal{L}_\pm$. So to the leading-order we get

$$\begin{aligned}&\int_0^1 dx\, x^{n-2}F_2^\gamma(x,Q^2,P^2) \ , \\ &= \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\sum_{i=\pm,NS}\mathcal{L}_i^n\frac{4\pi}{\alpha_s(Q^2)}\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right] \ ,\end{aligned} \tag{A·19}$$

which is nothing but the expression for the n_f light-flavor case as we expected. Note that in the above equation, we have used the fact:

$$\sum_{i=1}^{n_f-1}e_i^2+e_H^2=\sum_{i=1}^{n_f}e_i^2, \quad \sum_{i=1}^{n_f-1}e_i^4+e_H^4=\sum_{i=1}^{n_f}e_i^4 \ . \tag{A·20}$$

The above result means that there is no difference between the case with $n_f - 1$ light-flavor plus one heavy-flavor and the one with n_f light-flavors to the leading-order in QCD. For the NLO coefficients $\Delta\mathcal{A}_i^n, \Delta\mathcal{B}_i^n, \Delta\mathcal{C}^n$ we perform the similar analysis and get the additional contributions given in Eq. (2·18).

Appendix B

—— *Master formula for the moments in the parton picture* ——

There is an alternative method to derive the moment sum rule Eq.(2·17) which is based on the parton picture.[28] Consider the n-th moment of the virtual photon structure function $F_2^\gamma(x, Q^2, P^2)$ in the case where $n_f - 1$ light quarks and one heavy quark are present. The n-th moments of F_2^γ is given by

$$M_2^\gamma(n) = q_L^\gamma(n)C^L(n) + q_H^\gamma(n)C^H(n) + G^\gamma(n)C^G(n) + q_{NS}^\gamma(n)C^{NS}(n) + C^\gamma(n) \ , \tag{B·1}$$

where we have suppressed, for simplicity, the Q^2 as well as P^2 dependence of the moments of the structure function, the parton distributions and the coefficient functions. $q^\gamma_{L(NS)}$ denotes the flavor singlet (non-singlet) quark parton distribution function for the $n_f - 1$ flavors as defined in Eq.(2·4), and G^γ is the gluon parton distribution function. $C^i(i = L, H, G, NS, \gamma)$ are the coefficient functions for parton i-type in the virtual photon,

$$C^L(n) = \langle e^2\rangle_{n_f-1}\left(1 + \frac{\alpha_s}{4\pi}B_\psi^n\right), \tag{B·2}$$

$$C^{NS}(n) = 1 + \frac{\alpha_s}{4\pi}B_\psi^n, \tag{B·3}$$

$$C^H(n) = C^L + e_H^2\frac{\alpha_s}{4\pi}\Delta B_\psi^n, \tag{B·4}$$

$$C^G(n) = \langle e^4\rangle_{n_f-1}\frac{\alpha_s}{4\pi}B_G^n, \tag{B·5}$$

$$C^\gamma(n) = 2\beta_0\left\{\delta_\gamma B_\gamma^n + 3e_H^4(B_\gamma^n + \Delta B_\gamma^n)\right\}. \tag{B·6}$$

Putting all the above quantities together and noting that the heavy quark distribution differs from the light-flavor distribution by an extra contribution $\Delta q^{n_f}(n)$:

$$q^H(n) = q^{n_f}(n) + \Delta q^{n_f}(n) \ , \tag{B·7}$$

we obtain the moment which includes the heavy quark mass effects as

$$\begin{aligned} M^\gamma(n) = M^\gamma(n)\mid_{m=0} &+ e_H^2\Delta q^{n_f}(n) + 6\beta_0 e_H^4\Delta B_\gamma^n \\ &+ e_H^2\frac{\alpha_s}{4\pi}q^{n_f}(n)\Delta B_\gamma^n + e_H^2\frac{\alpha_s}{4\pi}G^\gamma(n)\frac{1}{n_f}\Delta B_G^n \ . \end{aligned} \tag{B·8}$$

Here we note that $\Delta B_\gamma^n = 2\Delta B_G^n/n_f$. Denoting $r = \alpha_s(Q^2)/\alpha_s(P^2)$ we have

$$\Delta q^{n_f}(n) = \frac{e_H^2 - \langle e^2 \rangle}{n_f(\langle e^4 \rangle - \langle e^2 \rangle^2)} \Delta q_{NS}^\gamma(n) + \frac{1}{n_f} \Delta q_S^\gamma(n) \ , \tag{B·9}$$

$$\Delta q_{NS}^\gamma(n) / \frac{\alpha}{8\pi\beta_0} = \Delta \mathcal{A}_{NS}^n (1 - r^{d_{NS}^n}) + \Delta \widetilde{C}_{NS}^n \ , \tag{B·10}$$

$$\Delta q_S^\gamma(n) / \frac{\alpha}{8\pi\beta_0} = \Delta \hat{\mathcal{A}}_S^{+n} (1 - r^{d_+^n}) + \Delta \hat{\mathcal{A}}_S^{-n} (1 - r^{d_-^n}) + \Delta \hat{C}_S^n \ , \tag{B·11}$$

where

$$\begin{aligned}
&\Delta \mathcal{A}_{NS}^n = -2\beta_0 \Delta A_n^{(2)NS}, \quad \Delta \hat{\mathcal{A}}_S^{\pm n} = -2\beta_0 \frac{\gamma_{\psi\psi}^{0,n} - \lambda_\mp^n}{\lambda_\pm^n - \lambda_\mp^n} \Delta A_n^{(2)\psi} \ , \\
&\Delta \widetilde{C}_{NS}^n = 2\beta_0 \Delta A_n^{(2)NS}, \quad \Delta \hat{C}_S^n = 2\beta_0 \Delta A_n^{(2)\psi} \ ,
\end{aligned} \tag{B·12}$$

with

$$\begin{aligned}
2\beta_0 A_n^{(2)NS} &= 12\beta_0 \widetilde{A}_{nG}^\psi (\langle e^4 \rangle - \langle e^2 \rangle^2) \ , \\
2\beta_0 A_n^{(2)\psi} &= 12\beta_0 \widetilde{A}_{nG}^\psi \langle e^2 \rangle \ .
\end{aligned} \tag{B·13}$$

The gluon distribution is given by

$$G^\gamma(n) / \frac{\alpha}{8\pi\beta_0} = \frac{4\pi}{\alpha_s} \mathcal{L}_G^{+n} (1 - r^{d_+^n + 1}) + \frac{4\pi}{\alpha_s} \mathcal{L}_G^{-n} (1 - r^{d_-^n + 1}) \ , \tag{B·14}$$

where

$$\mathcal{L}_G^{\pm n} = \frac{K_\psi^{0,n} \gamma_{G\psi}^{0,n}}{\lambda_\pm^n - \lambda_\mp^n} \frac{1}{1 + d_\pm^n}, \quad K_\psi^{0,n} = 24 n_f \langle e^2 \rangle_{n_f} \frac{n^2 + n + 2}{n(n+1)(n+2)} \ . \tag{B·15}$$

From these expressions we can derive the (2·17) with (2·18).

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